Materials and Structures (2019) 52:63
https://doi.org/10.1617/s11527-019-1363-8

ORIGINAL ARTICLE

An improved strut-and-tie model to predict the ultimate strength of steel fiber-reinforced concrete corbels

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Received: 27 February 2019 / Accepted: 25 May 2019 / Published online: 13 June 2019

Abstract In this paper, a strut-and-tie model is proposed for predicting the ultimate shear capacity of steel fiber reinforced concrete corbels. The proposed strut-and-tie model accounts for the effect of concrete strength, fiber volume, fiber aspect ratio, ratio of main steel and horizontal stirrups ratio, horizontal load ratio and shear span-to-depth ratio. The ultimate shear predictions of the proposed model are validated with 146 test results from the literature. The comparison shows that the proposed model performs well in predicting the ultimate shear capacity of steel fiber reinforced concrete corbels. The overall average value of the ratio between the experimental and the predicted strengths is 1.1 and the standard deviation is 0.105. Compared with the existing testing results, the strut-and-tie model predictions of the American and the British standard are more conservative than the improved strut-and-tie model. Also, comparative studies between the proposed model and the strut-and-tie models provided by other researchers in the literature are presented. Finally, sensitivity studies are performed for fiber parameters. The ratio between the ultimate shear strength for steel fiber reinforced concrete corbels and the ultimate shear strength for non-fibrous corbels are studied versus the steel fiber parameters considering shape of fiber and shear span-to-depth ratio. The studied fiber parameters are fiber volume content and fiber aspect ratio.

Keywords Corbels · Horizontal stirrups · Fiber aspect ratio · Steel fibers · Strut and tie · Ultimate shear capacity · Vertical load

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1 Introduction

Corbels are short cantilever members that stand out from a column or a wall to support another beam or heavy concentrated load. The importance of these members is clear in precast and industrial buildings where corbels support beams and girders. Corbels are characterized by a shear span-to-depth ratio (*a/d*) lower than unity. Two categories are defined for the failure of the concrete members, B-regions (Beam or Bernoulli) or D-regions (Disturbed or Discontinuity) [1, 2]. B-regions apply bending theory state that plane sections before bending remain plane after bending. Based on Bernoulli's hypothesis, it is easily to determine the internal forces from moment and shear diagrams satisfying equilibrium. In case of sudden change in geometry or a concentrated load is present in the concrete member, the strain distribution becomes nonlinear and the flow of internal forces is disturbed.

These regions are so-called D-Regions and the plane sections theory is no longer applicable for their design. Corbels transfer loads from primary beams and girders to the vertical elements. This means that corbels are subjected to high concentrated loads so that, corbel can be classified as D-region. For many years, researchers have been trying to find rational methods to analyze and design disturbed regions. Strut-and tie models (STM) have been developed and refined over the years to become a very powerful tool in the design and analysis of D-regions [1, 2].

Over the years, the contribution of steel fibers parameters has been studied on the structural behaviour of concrete corbels [3–13]. It was found that steel fibers could replace partially or fully the stirrups. In addition, using steel fiber improves the ductility and toughness of the reinforced concrete corbels. The aim of this paper is to present an analysis and design tool using strut-and-tie model (STM) for SFRC corbels [14]. The proposed STM accounts for the fibers contribution in concrete (compression and tension) and the composite tie action for the longitudinal steel and horizontal stirrups in order to predict the ultimate shear capacity for SFRC corbels. In addition, validation studies for the improved STM are made for 146 tested corbels from other researchers in the literature [3–13]. Furthermore, a comparative study between the improved STM, the ACI code [15] and the BS [16] is presented. Finally, comparative studies between the proposed model and the STM provided by other researchers [3, 17–19] is presented.

2 Mathematical formulation of the proposed strut-and-tie model

2.1 Main assumptions

The proposed strut-and-tie model (STM) in the current research is an enhancement model of the STM proposed in ACI Code 318-14 [15]. The following modifications are considered:

Enhanced concrete compressive strength due to existing of steel fiber;
The diagonal strut is prismatic shaped;
Tensile resistance is represented by composite tie action due to steel reinforcement, horizontal stirrups and steel fibers;

The strut softening factor (β_s) and the nodal zone stress condition factor (β_n) of normal concrete are replaced by the strut softening factor of SFRC (β_{sf}) and the nodal zone stress condition factor of SFRC (β_{nf}).

Figure 1 represents the geometrical properties of the proposed STM for the corbel [14]. As shown in the figure, vertical load (V_u) is applied at distance (a) from the column face with the horizontal load (N_u). The corbel under consideration can be idealized by primary tension horizontal top tie for the contribution of the tension steel (A_s), secondary tension horizontal ties for the contribution of the horizontal stirrups (A_h) and diagonal compression strut for the contribution of fibrous or non-fibrous concrete.

In order to establish the STM for the corbels as shown in Fig. 1, the corbel height, width and effective depth are denoted by (h), (b) and (d) respectively. The diameter of main bars and the total area are denoted by (ϕ) and (A_s) respectively. The STM is idealized as statically determinate truss with two main members as follows:

(1) Top horizontal tie (AC) with tension force ($F_{u,Tie}$) [Eq. (19)] and cross section area (A_{ct}) [Eq. (8)].

One diagonal strut (AB) with compression force ($F_{u,St}$) [Eq. (11)] and cross section area (A_{str}) [Eq. (7)].

2.2 Geometrical discretization of STM

The angle of inclination (θ) of the diagonal member as shown in Fig. 1 can be defined as:

$$\theta = \tan^{-1}\left(\frac{H}{a}\right) \tag{1}$$

The angle (θ) should be not less than 25° (degrees) according to ACI 318-14 [15].

Where a = The shear span measured from center of the bearing plate to the column face (mm); H = The distance between the acting compression force in concrete and the tension force in main steel (mm) and can be defined as:

$$H = d - \frac{Z}{3} \tag{2}$$

where Z = The depth of the neutral axis measured from the compression face (mm). Figure 2 shows the

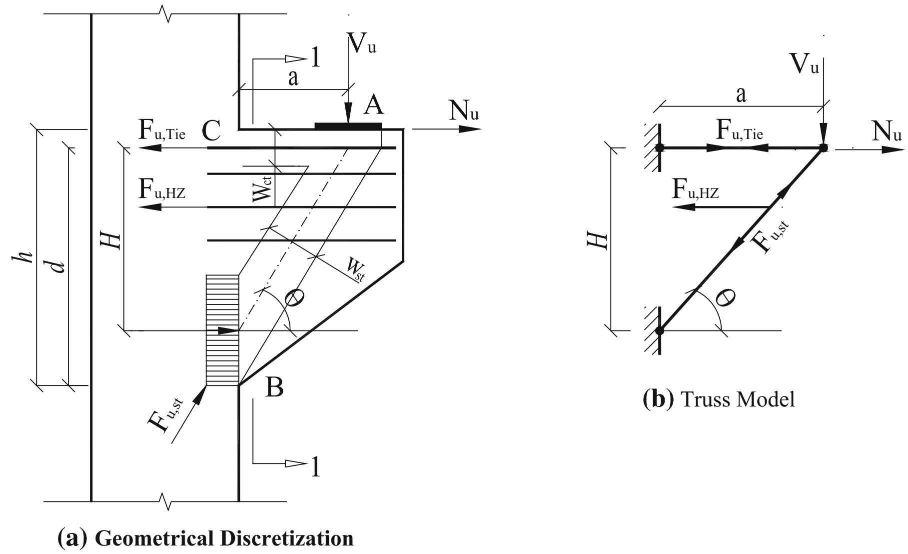

Fig. 1 Elements of the strut and tie model of SFRC corbel

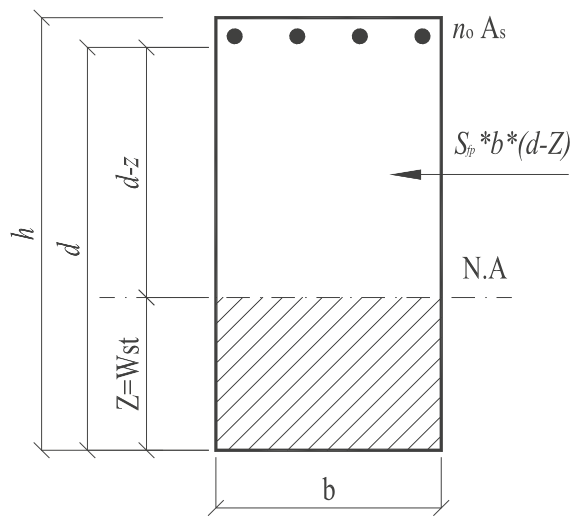

Fig. 2 Illustrative sketch for the depth of compression and tension zones

contribution of the main tensile steel reinforcement and the steel fiber in the tension zone, the depth of neutral axis (Z) can be calculated as:

$$\frac{bz^2}{2} = n_o A_s * (d - z) + S_{fp} * b * (d - z) \tag{3}$$

where n_o = The modular ratio of reinforcing steel = E_s/E_c; E_s = The Young's modulus of steel ($E_s = 2 \times 10^5$ MPa); E_c = The Young's modulus of concrete ($E_c = 4400*(f_c'^{0.5})$ MPa); S_{fp} = The steel fiber parameter and can be defined as [20]:

$$S_{fp} = m * \beta_o * V_f \tag{4}$$

where m = The modular ratio of the steel fibers = E_f/E_c; E_f = Young's modulus of the steel fiber ($E_f = 2.1 \times 10^5$ MPa); β_o = The effective orientation factor of the steel fiber assumed as 0.41 [21]; V_f = The volume percentage of the steel fibers.

In order to simplify Eq. (3), it can be rewritten as:

$$Z^2 + X_i * z - X_i * d = 0 \tag{5}$$

where X_i = Coefficient used to calculate the depth of the neutral axes and defined as:

$$X_i = 2 * \left(\frac{n_o * A_s}{b} + S_{fp} \right) \tag{6}$$

Then, the term (A_{str}) is assumed to be the cross-sectional area of the diagonal compressive strut (AB), while (A_{ct}) is considered as the cross-sectional area of top tie (AC) and can be expressed as:

$$A_{str} = b * w_{st} \tag{7}$$

$$A_{ct} = b * w_{ct} \quad (8)$$

where w_{st} = The depth of the diagonal compressive strut (mm) = (the depth of the neutral axis (Z)); w_{ct} = The depth of the top tension tie (mm), and is defined as:

$$w_{ct} = 2d' + \phi(\text{used}) \quad (9)$$

In case of providing horizontal stirrups in the corbel, it can be idealized by horizontal ties with resisting force ($F_{u,HZ}$) has depth of ties (w_{cth}) for each stirrup that can be assumed as [14]:

$$w_{cth} = \phi_{hz} + 2 * d' \quad (10)$$

where d' = The concrete cover (mm); ϕ_{hz} = The diameter of the used horizontal stirrups in the corbel (mm).

2.3 Strength of SFRC compression strut

The compression capacity of the diagonal strut ($F_{u,st}$) can be estimated depending on the shape of strut which is calculated generally as:

$$F_{u,st} = f_{cd} * A_{str} \quad (11)$$

where f_{cd} = effective compressive strength of fibrous concrete strut, it can be defined as:

$$f_{cd} = \alpha * \beta_{sf} * f'_{cf} \quad (12)$$

where α = Flexure coefficient depend on the design code. (Considered as 0.85 according to ACI code [15]); f_{cf}' =cylindrical compressive strength of the fibrous concrete, and can be defined as [22]:

$$f'_{cf} = f'_c * (1 + 0.1066 * F) \quad (13)$$

where f_c'= cylindrical compressive strength of non-fibrous concrete; F =fiber factor, it can be defined as [22]:

$$F = V_f * \frac{l_f}{\phi_f} * \lambda \quad (14)$$

where l_f = The fiber length (mm); ϕ_f = The fiber diameter (mm); λ = The fiber shape factor; λ = 1.0 in case of hocked end steel fiber; λ = 0.50 in case of straight steel fiber; β_{sf}= The strut softening factor of fibrous concrete and can be defined as [23];

$$\beta_{sf} = \beta_s + 0.28F \quad (15)$$

where β_s = The strut softening factor for non-fibrous concrete and can be determined according to the shape of strut [15] (β_s = 0.7 for C–C–T Joint).

Conclusively, the compression capacity of the diagonal strut ($F_{u,st}$) can be defined as:

$$F_{u,st} = 0.85 * \beta_{sf} * f'_{cf} * b * w_{st} \quad (16)$$

2.4 Composite tie strength of tension steel and horizontal stirrups

An equivalent tension member is considered having steel area embedded in the tension zone. The proposed tie is composite of the strength of the reinforcing steel and surrounding by fibrous concrete concentric with the axis of the tensile force. The composite tie is shown in Fig. 3 and is indicated with the finer hatched area. The effective area of composite tie is given by;

$$A_s^{eff} = n_s * (w_{ct})^2 \quad (17)$$

Then

$$A_s^{eff} = n_s * (2d' + \phi)^2 \quad (18)$$

where n_s = The number of main longitudinal top bars adopted for tension steel. The tensile strength of a composite tie ($F_{u,Tie}$) is taken as:

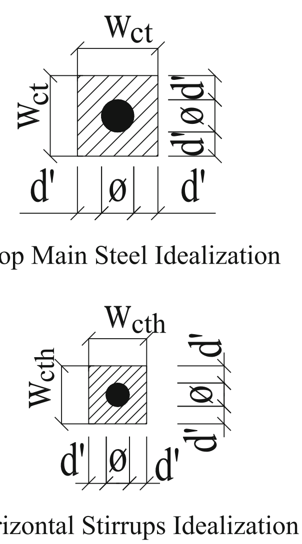

Fig. 3 Composite details for tension steel and horizontal stirrups

$$F_{u,Tie} = f_y * A_s + \sigma_{pc} * \left(A_s^{eff} - A_s\right) \quad (19)$$

where A_s = The reinforcing area of steel bars in tension composite tie, and can be provide as:

$$A_s = n_s * A_{bar} \quad (20)$$

where A_{bar} = The area of one steel bar (mm^2); f_y = The yielding stress of the steel bars (MPa); σ_{pc} = The post-cracking tensile fibrous concrete strength (MPa) as defined by [24]:

$$\sigma_{pc} = 0.2872F\left(f'_{cf}\right)^{\frac{2}{3}} \quad (21)$$

Accordingly, the tension force in the top composite tie ($F_{u,tie}$) can be calculated as:

$$F_{u,Tie} = n_s \cdot \left[\left(f_y * A_{bar}\right) + \sigma_{pc}\left(\left(w_{ct}\right)^2 - A_{bar}\right)\right] \quad (22)$$

Generally, stirrups close to the main steel reinforcement at the maximum tension zone tend to reach the yield strength level (f_{yh}) rather than the stirrups away from the maximum tension zone. Also, some stirrup layers could even be ineffective in tension side. Consequently, it can be assumed initially that the mean tensile stress f_{shm} in the stirrups is equal to ψf_{yh}. Noting that the value of ψ is considered as 0.5 [17]. Then, the force in the composite horizontal stirrups ($F_{u,HZ}$) is given by;

$$F_{u,HZ} = n_h \cdot \left[\left(\psi * f_y * A_{bar\cdot st}\right) + \sigma_{pc}\left(\left(w_{cth}\right)^2 - A_{bar\cdot st}\right)\right] \quad (23)$$

where n_h = The number of the horizontal stirrups adopted for tension steel; ψ = The coefficient to calculate mean tensile stress in horizontal stirrups = 0.5 [17]; $A_{bar\cdot st}$= The area of one horizontal stirrup (mm^2); w_{cth}= The width and the depth of the composite horizontal tie for each horizontal stirrups (mm).

2.5 Derivation of shear carrying capacity

With reference to the truss shown in Fig. 1, the nodal zone (A) is idealized in the STM for the corbel as compression–compression–Tension joint ((C–C–T) Joint). The equilibrium conditions give the axial force ($F_{u,Tie}$) in the main steel for tie (AC), ($F_{u,HZ}$) for horizontal stirrups and in the concrete strut member (AB) ($F_{u,st}$) expressed respectively by; Summation of the horizontal forces at joint A = 0 Then;

$$F_{u,Tie} + F_{u,HZ} = \frac{V_u}{\tan\theta} + N_u$$

(Force in the composite tie)

$$\therefore V_u = \left(F_{u,Tie} + F_{u,HZ} - N_u\right)\tan\theta \quad (24)$$

Then V_{u1} = V_u is the shear strength related to the yielding of the main bars.

$$F_{u,st} - \frac{F_{u,HZ}}{\cos\theta} = \frac{V_u}{\sin\theta} - \frac{N_u}{\cos\theta}$$

(Force in the compression strut)

$$\therefore V_u = \left[F_{u,st} - \left(F_{u,HZ} - N_u\right)/\cos\theta\right]\sin\theta \quad (25)$$

Then V_{u2} = V_u is the shear strength related to the failure of the compression strut (neglecting the effect of the tie).

Where V_u = The applied vertical load on the corbel (kN); N_u = The applied horizontal load on the corbel (kN); it can be given by;

$$N_u = \Delta * V_u \quad (26)$$

Δ = ratio of the horizontal force to the vertical force acts on the corbel, ranged from 0 to 0.2 of V_u according to ACI 318-14 [15].

Substituting the data given in Eqs. (22) and (23) into Eq. (24):

$$V_{u1} = \left[n_s\left[\left(f_y * A_{bar}\right) + \sigma_{pc}\left(\left(w_{ct}\right)^2 - A_{bar}\right)\right] + n_h \cdot \left[\left(\psi * f_y * A_{bar\cdot st}\right) + \sigma_{pc}\left(\left(w_{cth}\right)^2 - A_{bar\cdot st}\right)\right] - N_{u1}\right] * \tan\theta \quad (27)$$

Substituting the data given in Eqs. (16) and (23) into Eq. (25):

$$V_{u2} = \left[\left[0.85 * \beta_{sf} * f'_{cf} * b * w_{st}\right] - \frac{n_h \cdot \left[\left(\psi * f_y * A_{bar\cdot st}\right) + \sigma_{pc}\left(\left(w_{cth}\right)^2 - A_{bar\cdot st}\right)\right]}{\cos\theta} + \frac{N_{u2}}{\cos\theta}\right] * \sin\theta \quad (28)$$

Then the shear carrying capacity (V_u) is the smaller value from Eqs. (27) and (28).

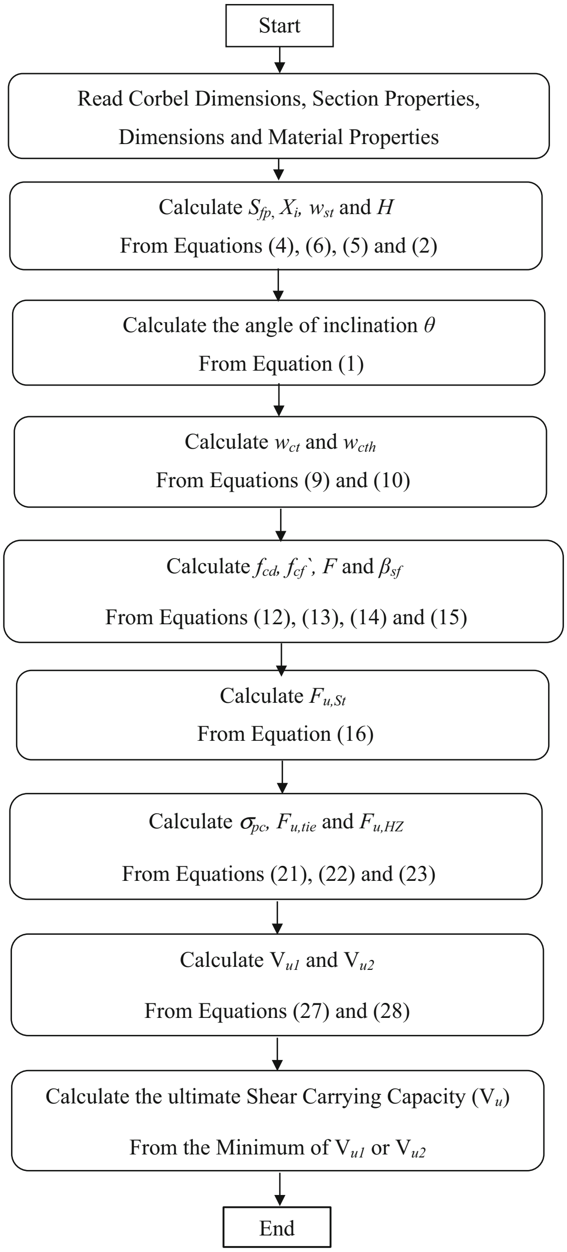

Fig. 4 Implementation flow chart for the ultimate shear strength of SFR corbels

3 Model implementation and validation studies

Figure 4 shows the flow chart used to calculate the ultimate shear strength (V_u) of the SFRC corbels using the proposed STM. The procedure of the improved (STM) for the steel fiber reinforced concrete corbels can be easily implemented by hand calculations or a spreadsheet. 146 reinforced concrete corbels tested by other researchers [3–13] have been analyzed by the proposed STM model [14]. These corbels having different fiber volume ranging from 0 to 2.5%, an overall depth ranging from 150 to 600 mm, the main longitudinal reinforcement ratios ranged from (0.22 to 3.4%), having variables percentages of the horizontal stirrups ranging from 0 to 1.77% and shear span-to-depth ratio (a/d) ranged from 0.25 to 1.45. The concrete cylindrical strengths (f_c') ranged from 20.7 to 64 MPa. The ultimate experimental shear strength ($V_{u(EXP)}$) and the predicted ultimate shear strength by the proposed STM ($V_{u(STM)}$) is plotted in Fig. 5.

As shown in Fig. 5, the improved STM for steel fiber reinforced concrete corbels generally performs well in predicting the ultimate shear strengths. The overall average value of the ratio between the experimental strength and the predicted shear strength of the proposed STM [$V_{u(EXP)}/V_{u(MSTM)}$] is of value 1.10 with a standard deviation of 0.105.

4 Comparative studies with the design codes and previous models

The relationships between the ultimate experimental shear strength and the ultimate shear strength from the design codes [15, 16] and the previous models [3, 17–19] are plotted in Figs. 6, 7 and 8. It has to be noted that the effect of steel fiber contribution in the ultimate shear strength of the SFRC corbels is not accounted in the design codes equations of ACI [15]

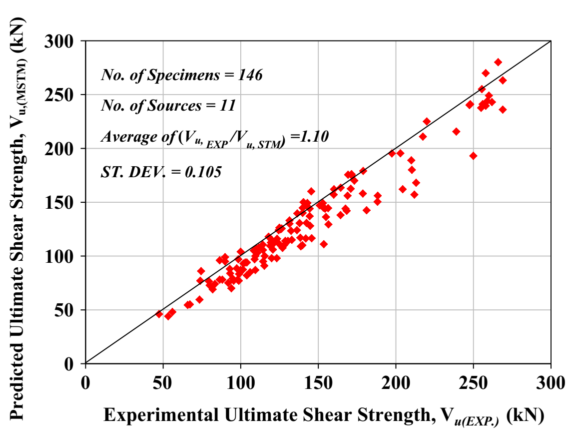

Fig. 5 Comparison of the ultimate shear strength predictions using the proposed STM [14] and the experimental results

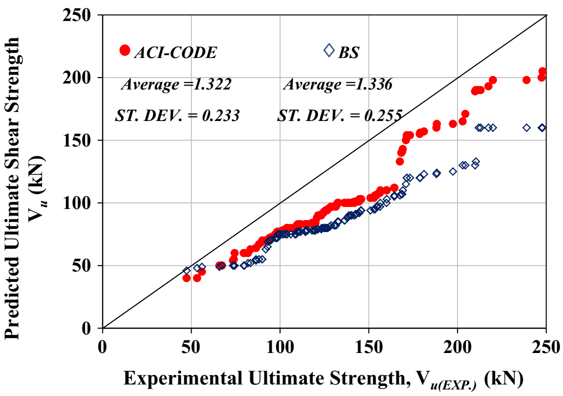

Fig. 6 Comparison of the ultimate shear strength predictions using STM of ACI code [15] and BS [16] with the experimental results

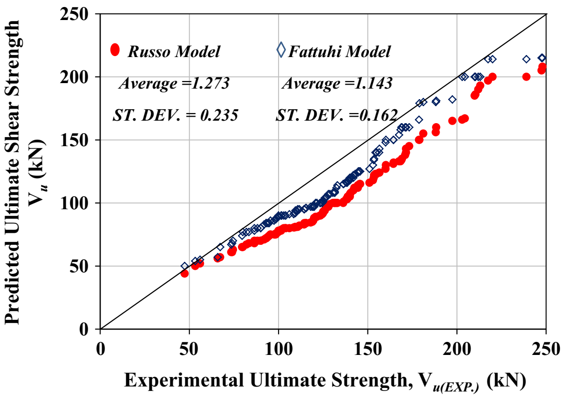

Fig. 7 Comparison of the ultimate shear strength predictions using STM of Russo [17] and Fattuhi [3] with the experimental results

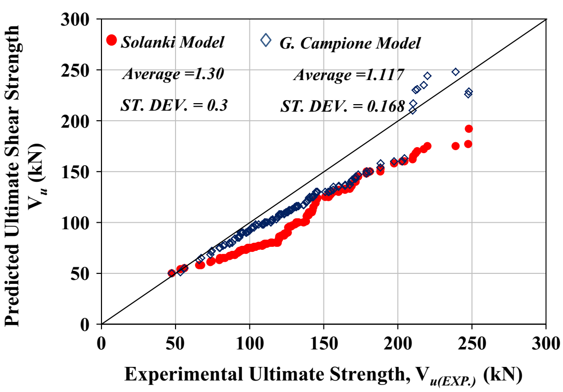

Fig. 8 Comparison of the ultimate shear strength predications using STM of Solanki [18] and G. Campione [19] with the experimental results

and BS [16]. Compering with the design codes, the overall average values of the ratio between the experimental shear strength and the predicted shear strength [$V_{u(EXP)}/V_{u(ACI)}$] and [$V_{u(EXP)}/V_{u(BS)}$] are 1.322 and 1.336 respectively. Compering with the previous models, the overall average values of the ratio between the experimental ultimate shear strength and the predicted ultimate shear strength [$V_{u(EXP)}/V_{u(Russo)}$], [$V_{u(EXP)}/V_{u(Fattuhi)}$], [$V_{u(EXP)}/V_{u(Solanki)}$] and [$V_{u(EXP)}/V_{u(G.\ Campione)}$] are 1.273, 1.143, 1.299 and 1.117 respectively.

Based on the predicted results, the ACI Code [15] and the BS [16] are more conservative than the improved STM. Russo model [17] has variation corresponding to the proposed STM by 16%; the influence of steel fiber parameters in the improved STM improves the predicted ultimate shear capacity corresponding to the experimental results. Good correlation is observed between Fattuhi empirical equation [3] and the proposed STM, 4.20% variation is observed from the comparison between the results. An enhancement in the results is obtained by the improved STM with respect to Solanki model [18] by 18.4%. By comparing the results calculated by the proposed STM by G. Campione model [19], the results are almost the same; only 1.8% variation is observed from the comparison. The proposed model yields better predictions than the models proposed by other scholars or available methods in the codes, with the added benefit of giving the designer the possibility of optimizing fiber amount in terms of aspect ratio and content by volume, for different values of the span-to-depth ratio of the corbel.

5 Sensitivity studies for fiber parameters

Using the proposed STM model, sensitivity studies are performed to study the effect of fiber parameters on the ratio between the shear strength for SFRC corbels ($V_{u,SFRC}$) and the shear strength for RC corbels ($V_{u,RC}$). The studied parameters are fiber volume content (V_f%) and fiber aspect ratio (l_f/ϕ_f) as shown in Figs. 9 and 10 respectively. The ratio [$V_{u,SFRC}/V_{u,RC}$] is plotted versus the fiber parameters considering the shape of fiber (Hooked end and straight) and shear span-to-depth ratio (*a/d*).

From Fig. 9, it can be concluded that the ultimate shear strength of SFRC corbels ($V_{u,SFRC}$) is improved

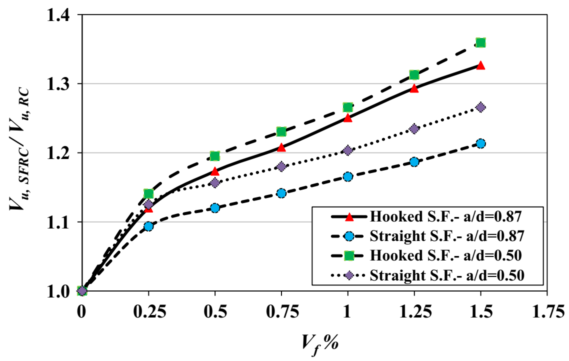

Fig. 9 Effect of fiber volume (V_f) on the shear strength predictions

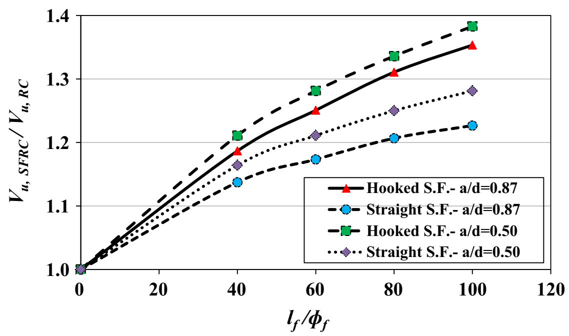

Fig. 10 Effect of fiber aspect ratio (l_f/ϕ_f) on the shear strength predictions

by increasing fiber volume (V_f), decreasing shear span-to-depth ratio (*a/d*) and using hooked end steel fibers in lieu of straight fibers. Considering constant (*a/d*) ratio as 0.87, the increase of (V_f) from 0.0 to 1.5% enhances the $V_{u,SFRC}$ by 20% for corbel with straight fiber and by 29% for corbel with hooked end fiber. Keeping the hooked fiber volume as 1.0%, the decrease of (*a/d*) ratio from 0.87 to 0.5 improves the ultimate shear strength ($V_{u,SFRC}$) by 45%. As presented in Fig. 10, the ultimate shear strength of SFRC corbels ($V_{u,SFRC}$) is improved by increasing fiber aspect ratio (l_f/ϕ_f) for different values of shear span-to-depth ratio. Considering constant (*a/d*) ratio as 0.5, the increase of (l_f/ϕ_f) from 0.0 to 100 enhances the $V_{u,SFRC}$ by 29% for corbel with straight fiber and by 48% for corbel with hooked end fiber.

6 Conclusions

In the paper, a strut-and-tie model is proposed for predicting the ultimate shear capacity of steel fiber reinforced concrete corbels. The proposed model accounts for the effect of concrete strength (f_c'), fiber volume (V_f), fiber aspect ratio (l_f/ϕ_f), ratio of main steel (ρ_s) and horizontal stirrups ratio (ρ_h), horizontal load ratio (N_u) and shear span-to-depth ratio (*a/d*). Based on the predicted ultimate shear strength by the model and the results of 146 tested corbels from the literature, the following conclusions can be drawn:

(1) The proposed STM for SFRC corbels proves to be successful as a design and analytical tool for predicting the ultimate shear capacity (V_u) of SFRC corbels. The STM predictions for 146 experimental results are on the safe side and gives consistent predictions [14]. The overall average value of the ratio [$V_{u,\ EXP}/V_{u,STM}$] is 1.10 and a standard deviation 0.105.

(2) Compared with the existing experimental test results, STM predictions of (ACI) code [15] and (BS) [16] are more conservative than the improved STM. The overall average ratio between the experimental load and the predicted capacity for ACI [15] and BS [16] are 1.32 and 1.336 design codes respectively.

(3) Comparing the experimental test results with the existing STM in the literature indicates that the proposed STM is a good tool in the analysis of SFRC corbels. The overall average value of the ratios [$V_{u,EXP}/V_{u,STM}$] for Russo model [17] and G. Campione model [19] are 1.273 and 1.117 respectively.

(4) The parametric studies of the proposed model indicate that the ultimate shear strength of SFRC corbels is improved by increasing fiber volume (V_f), increasing fiber aspect ratio, decreasing shear span-to-depth ratio (*a/d*) and using hooked end steel fibers in lieu of straight fibers. Comparing with non-fibrous concrete corbels, the improvement in shear strength is 29% due to fiber volume inclusion as 1.5%, and is 49% due to fiber use with aspect ratio as 100.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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